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A NUMERICAL ALGORITHM FOR THE ANALYSIS OF THE THERMAL STRESS-STAIN STATE OF A ROD

Annotation— This article deals with the development of a mathematical model, appropriate computational algorithms and a set of application programs in a high-level object-oriented programming language, which allow us to numerically simulate and study the thermo mechanical state of rods, while having local thermal insulation, heat exchange, temperature and axial forces, taking into account pinching the ends of the rod.

Keywords— universal algorithm; thermo elastic state; limited length; axial force; heat flow; efforts of the rod; axial tensile force.

INTRODUCTION

Studies of the thermo mechanical state of rod-bearing structural elements with the simultaneous presence of axial forces, local thermal insulation, heat exchange and temperature, which can be constant, varying along the local rod length by a linear and quadratic law, are of particular interest in many technological processes ensuring the thermal strength of structural elements that work in a complex heat and force field.

The development of modern competitive internal combustion engines, gas turbine power plants, oil heating compressor stations, steam generators of technological processes, which enable deep processing of uranium and osmium ores, as well as crude oil, confront current scientists with the development of mathematical models, appropriate computational algorithms, methods, and a set of applied programs allowing to numerically investigate the thermo mechanical state of bearing elements these designs, taking into account their operating conditions.

It should be noted the complexity of the study of the thermo mechanical state of rods of limited length, with the simultaneous presence of axial forces, local thermal insulation, heat exchange and temperatures, which is specified in different forms.

CALCULATION SCHEME OF THE TASK

Suppose a rod of limited length is given, both ends of which are rigidly clamped. The cross-sectional area is constant along its length. The heat flux is supplied to the portion $x_1 \le x \le x_2$ of the side surface of the rod through the cross-sectional area, which correspond to points $(x=0)$ and $(x=L)$ heat exchange takes place with their environment. Here, respectively, the heat transfer coefficient will be $h_0 \left(W/cm^{2\,o}C\right)$

and $h_L\left(W/cm^2\,{}^oC\right)$, the ambient temperature $T_0(^0C)$ and $T_L(^0C)$ (figure 1). The rest of the side surface of the rod, i.e. plots $0 \le x \le x_1$ and $x_2 < x \le x_L$ insulated.

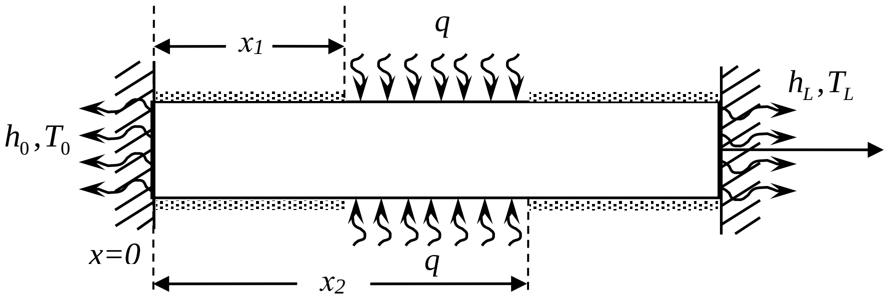

Figure-1 - Calculation scheme of rod extension

As a result, the heat flux is supplied to the side surface of the rod and heat exchange through the cross-sectional area with their environment causes a temperature field along the length of the rod.

Due to the fact that both ends of the rod are rigidly clamped, a compressive force arises in the rod and as a result a compressive stress appears. Now it is necessary to investigate the regularity of the dependence of these values on the heat flux, heat transfer coefficient and ambient temperature. To do this, first, given the boundary conditions, it is necessary to find the temperature distribution field along the length of the rod. For this, the length of the rod is divided into n equal parts. Then the length of one part will be $l = L/n$. Now we take one element of the rod, the length of which $l\,(cm)$. We consider this element as a quadratic finite element. In this element we take three nodes i, j and k. Wherein $x_j - x_i = x_k - x_j$. If we consider the temperature distribution field within an element such as a second-order curve passing through three points, then within a given element its expression will be as follows

$$T(x) = \varphi_i(x)T_i + \varphi_j(x)T_j + \varphi_k(x)T_k, \quad x_i \le x \le x_k, \tag{1}$$

Now for the first finite element $(0 \le x \le l)$ we will write a functional expressing thermal energy

$$I_1 = \int_{V_1} \frac{K_{xx}}{2}\left(\frac{\partial T}{\partial x}\right)^2 dV + \int_{S_1} \frac{h_0}{2}(T - T_0)^2 dS\ , \tag{2}$$

where V_1-is the volume of the first element, S_1 – the cross-sectional area of the first element corresponding to the point $x = 0$.

Now consider the section $x_1 \le x \le x_2$ of the rod. Due to the fact that the heat flux is supplied to the side surface of this section of the rod, for the finite elements of this section the expression of the thermal energy functional has the following form

$$I_i = \int_{V_i} \frac{K_{xx}}{2}\left(\frac{\partial T}{\partial x}\right)^2 dV + \int_{S_{alsn}^{(i)}} qTdS \qquad (3)$$

where $i = ((x_1 / l + 1) \div x_2 / l)$; $S_{als}^{(i)}$ – is the area of the lateral surface of the i-th finite element.

Now consider the last n-finite element of the rod. Due to the fact that through the cross-sectional area of this element, which corresponds to a point $x = L$, heat exchange occurs with the environment and the heat exchange coefficient h_L, the ambient temperature T_L, then for this element the expression of the thermal energy functional has the following form

$$I_n = \int_{V_n} \frac{K_{xx}}{2}\left(\frac{\partial T}{\partial x}\right)^2 dV + \int_{S_L} \frac{h_L}{2}(T - T_L)^2 dS , \qquad (4)$$

where S_L – is the cross-sectional area corresponding to the point $x = x_L$.

So in the rod under consideration, the number of finite elements will be n, therefore for the rod as a whole, the expression of the thermal energy functional has the following form

$$I = \sum_{i=1}^{n} I_i \ . \qquad (5)$$

In each finite element, the number of nodal points is 3, so the number of nodal points along the length of the rod will be $(2n+1)$. Then, to determine the temperature in these nodes, we minimize the functional (5) from the node temperature values and obtain the following system of linear algebraic equations

$$\frac{\partial I}{\partial T_i} = 0, \ \ i = 1,2,....(2n+1) \, . \qquad (6)$$

Solving the resulting system by the Gauss method, the temperature values at the nodal points of the finite elements are determined. And then the law of distribution of the temperature field $T = T(x)$ along the length of the rod is constructed.

Using this, we begin to look for the law of distribution of elastic displacements, components of deformations and stresses, as well as the value of thermo elastic stress and compressive force along the length of the rod. For this, we consider the rod under consideration by $m = n/2$-quadratic finite elements of the same length. Then the number of nodal points of finite elements will be $(2m+1)$. For any i finite element, the expression of potential energy will be as follows

$$\Pi_i = \int_{V_i} \frac{\sigma_x \varepsilon_x}{2} dV - \alpha E \int_{V_i} T(x) \varepsilon_x dV \, , \qquad (7)$$

where

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial \varphi_i(x)}{\partial x} T_i + \frac{\partial \varphi_j(x)}{\partial x} T_j + \frac{\partial \varphi_k(x)}{\partial x} T_k, \ \sigma_x = E\varepsilon_x \qquad (8)$$

Due to the fact that both ends of the rod are rigidly clamped, these points do not move. Then there will be $u_1 = u_{2m+1} = 0$. Then the form of the functional expressing potential energy for the considered rod will be as follows

$$\Pi_i = \sum_{i=1}^{m} \Pi_i \ , \qquad (9)$$

Next, by minimizing the total potential energy of the rod under consideration from the nodal values of displacements other than the 1st and the $(2m+1)$ nodes, to determine them we obtain the following system of linear algebraic equations

$$\frac{\partial \Pi}{\partial u_i} = 0, \ i = 2 \div 2m \qquad (10)$$

Solving the resulting system by the Gauss method, we determine the value of the elastic displacements of the nodal points of finite elements. Then we construct the distribution law of the field of elastic displacements $u = u(x)$ along the length of the rod. Then using expression (8) we construct the distribution field of elastic strains and stresses. And the temperature stress distribution field is constructed using an expression $\sigma = -\alpha E T(x)$.

THE ANALYSIS OF THE EFFECT

After approbation of the developed computational algorithm developed, in this example we analyze the effect of heat flow on the thermally deformed state of the rod under study. To do this, we calculate the values of the compressive force $R, (kg)$ and the true stress $\sigma, \left(kg/cm^2\right)$ at different values of the rod length. These results are shown in figure-2,3,4,5 and table 1.

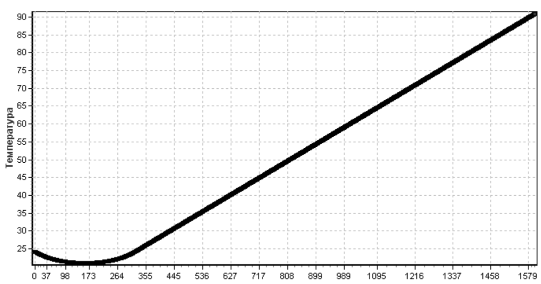

Figure-2 - The field of temperature distribution along the rod with $q = -70(W/cm^2)$

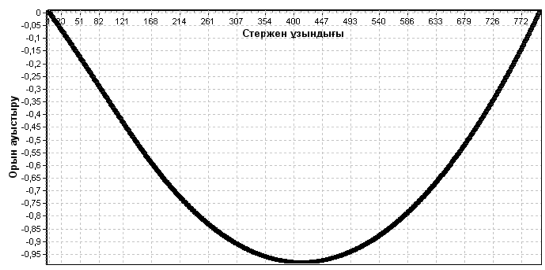

Figure-3 - The law of distribution of displacements of nodal points along the length of the rod with $q = -70(W/cm^2)$

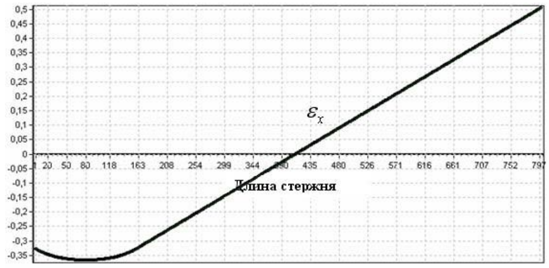

Figure-4 - The distribution field ε_x along the length of the rod with $q = -70(W/cm^2)$

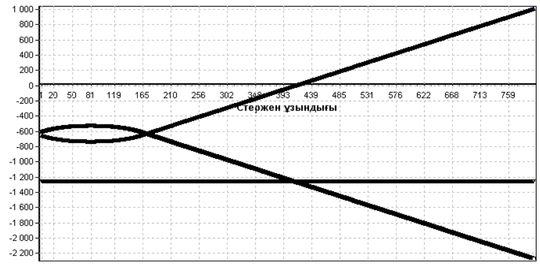

Figure-5 - The distribution field $\sigma_x, \sigma_T, \sigma$ along the length of the rod with $q = -70(W/cm^2)$

Table 1- The effect of heat flux on the thermal stress-strain state of the rod under study

№	Участки стержня	$R,(kg)$	$\sigma,(kg/cm^2)$	%
1	$0 \le x \le 16(cm)$	-133915	-6695,74	100
2	$16 \le x \le 32(cm)$	-20544	-9610,70	143,53
3	$32 \le x \le 48(cm)$	-208018	-10411,89	155,50
4	$48 \le x \le 64(cm)$	-181986	-9099,30	135,89
5	$64 \le x \le 80(cm)$	-113459	-5672,94	84,40

CONCLUSION

An appropriate computational algorithm has been developed for the numerical study of a thermally strained state of a partially heat-insulated and clamped by two ends of a rod in the presence of local temperature and heat transfer. A numerical study was conducted with different initial data.

In numerical experiments in this problem, it was found that at a given temperature in the areas $0 \le x \le 16(cm)$, $16 \le x \le 32(cm)$, $32 \le x \le 48(cm)$, $48 \le x \le 64(cm)$ and $64 \le x \le 80(cm)$ the rod, the value ε_x will be respectively *100; 82.2; 95.96; 102.61* and *117.10%.* And also the value $(\sigma = \sigma_x + \sigma_T)$ will be respectively *100; 104.47, 101.27, 99.17* and *94.60%.* And here the value of the compressive force of the rod will be respectively 100; *102.38* and *101.26%.*

An appropriate computational algorithm has been developed for the numerical study of the force of a partially heat-insulated rod clamped by two ends in the presence of heat flux.

At the same time, it was revealed that when the heat flux is supplied in the $0 \le x \le 16(cm)$, $16 \le x \le 32(cm)$, $32 \le x \le 48(cm)$, $48 \le x \le 64(cm)$ and $64 \le x \le 80(cm)$ rod sections, the values of the deformation components and the component stresses will change accordingly ε_x = *100; 138,18; 148,18;130,08 и 83,81%,* σ_T = *100; 141,64; 152,93;133,85 и 84,40%,* σ_x = *100; 138,20; 148,18; 130 и 83,80%,* σ =*100; 143,53; 155,50;135,89 и 84,72%.*

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